APPLICATION OF BOUNDARY EIGENSOLUTION CONCEPTS IN COMPUTATIONAL MECHANICS

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The classical eigenproblems of engineering mechanics are formed by inserting an eigenparameter into the governing differential equation and then enforcing homogeneous boundary conditions. For example, we may consider a scalar problem governed by the Laplace equation. The corresponding classical eigenproblem becomes

$$\nabla^2 u + \lambda u = 0 \qquad \text{in } V \tag{1a}$$

with

$$u = 0 \quad \text{on } S_u, \qquad q = 0 \quad \text{on } S_q \tag{1b,c}$$

for potential u and normal flux q. Notice however that the eigenfunctions of (1) are not, in general, harmonic. Instead these eigensolutions are related to the natural frequencies and mode shapes.

Alternatively, we may keep the governing partial differential equation intact, but introduce the eigenparameter λ and a positive weighting function ϕ into the boundary condition. Thus,

$$\nabla^2 u = 0 \qquad \text{in } V \tag{2a}$$

with

$$q = \lambda \phi u \qquad \text{on } S \tag{2b}$$

The eigensolutions of (2) also have very interesting properties. For example, all of the eigenvalues are real and non-negative. Additionally, the associated harmonic eigenfunctions are orthogonal over the boundary S and can be used in a generalized Fourier series to represent solutions to the Laplace equation with arbitrary well-defined boundary conditions.

More detail on the theory of boundary eigensolutions can be found in References [1,2], while initial applications in computational mechanics are discussed in References [3,4]. In this presentation, we focus on more recent applications, including the accurate evaluation of generalized stress intensity factors for non-smooth problems, a boundary stress algorithm for use within the finite element method, new formulations for interfacing boundary elements and finite elements, and a conformal mapping-based finite element approach.

References

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